SOLAR NEUTRINOS:WHAT NEXT?

J. N. BAHCALL[a]

[a]Institute for Advanced Study, Princeton, NJ 08540, USA
E-mail: jnb@sns.ias.edu

I summarize the current state of solar neutrino research and then give my answer to the question: What should we do next?

1. Introduction

The reader who is familiar with solar neutrino research may wish to skip directly to the last section entitled: What Next?

Solar neutrinos have been detected experimentally with fluxes and energies that are qualitatively consistent with solar models that are constructed assuming that the sun shines by nuclear fusion reactions. The first experimental result, obtained by Ray Davis and his collaborators in 1968, [1,2] has now been confirmed by four other beautiful experiments, Kamiokande, [3] SAGE, [4] GALLEX, [5] and SuperKamiokande. [6] The observation of solar neutrinos with approximately the predicted energies and fluxes establishes empirically the theory [7] that main sequence stars derive their energy from nuclear fusion reactions in their interiors and has inaugurated what we all hope will be a flourishing field of observational neutrino astronomy.

Although the calculated neutrino fluxes depend upon high powers of the central temperature of the solar model, the experiments and the solar model theory are so precise that persistent quantitative discrepancies have existed between the model predictions and the solar model calculations for over thirty years [8–10]

Important experiments are underway that will provide diagnostic information about the physical properties of neutrinos that are created in the center of the sun and detected on earth in really long baseline experiments. At this workshop, we will hear discussions of the SuperKamiokande, SNO, BOREXINO, HELLAZ, HERON, ICARUS, LENS, and KamLAND experiments.

I will discuss predictions of the combined standard model in the main part of this review. By 'combined' standard model, I mean the predictions of the standard solar model and the predictions of the standard electroweak model. We need a solar model to tell us how many neutrinos of what energy are produced in the sun and we need an electroweak theory to tell us how the number and flavor content of the neutrinos are changed as they make their way from the center of the sun to detectors on earth. For all practical purposes, the standard electroweak model states that nothing happens to solar neutrinos after they are created in the deep interior of the sun. Using standard electroweak theory and fluxes from the standard solar model, one can calculate the rates of neutrino interactions in different terrestrial detectors with a variety of energy sensitivities. The combined standard model also predicts that the energy spectrum from a given neutrino source should be the same for neutrinos produced in terrestrial laboratories and in the sun and that there should not be measurable time-dependences (other than the seasonal dependence caused by the earth's orbit around the sun). The spectral and temporal departures from standard model expectations are expected to be small in all currently operating experiments [11] and have not yet yielded definitive results. Therefore, I will concentrate here on inferences that can be drawn by comparing the total rates observed in solar neutrino experiments with the combined standard model predictions.

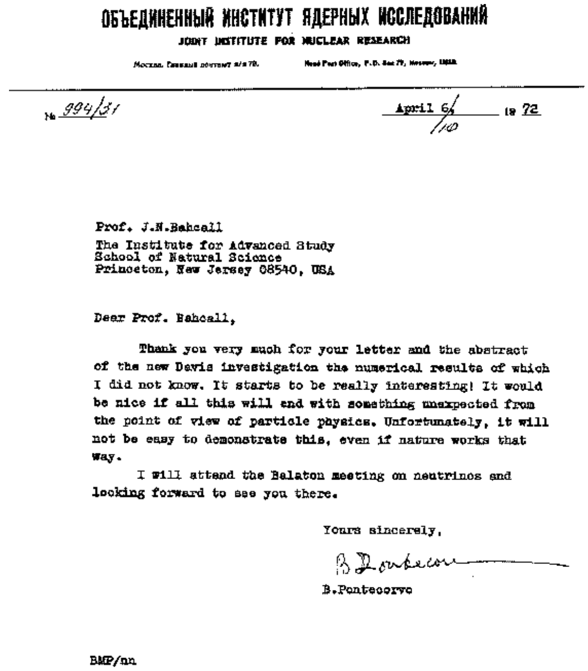
I will begin by reviewing in Section 2 the quan-

titative predictions of the combined standard solar model and then describe in Section 3 the three solar neutrino problems that are established by the chlorine, Kamiokande, SAGE, GALLEX, and SuperKamiokande experiments. In Section 4, I detail the uncertainties in the standard model predictions and then show in Section 5 that helioseismological measurements indicate that the standard solar model predictions are accurate for our purposes. In Section 5, I discuss the implications for solar neutrino research of the precise agreement between helioseismological measurements and the predictions of standard solar models. Next, ignoring all knowledge of the sun, I cite analyses in Section 6 that show that one cannot fit the existing experimental data with neutrino fluxes that are arbitrary parameters, unless one invokes new physics to change the shape or flavor content of the neutrino energy spectrum. I summarize in Section 7 the characteristics of the best-fitting neutrino oscillation descriptions of the experimental data. Finally, I will discuss and summarize the results in Section 8.

If you want to obtain numerical data or subroutines that are discussed in this talk, or to see relevant background information, you can copy them from my Web site: http://www.sns.ias.edu/∼jnb.

Before we begin the detailed discussion, I want to make just a brief historical diversion. Nearly all of the current interest in solar neutrinos centers around the opportunity to use the sun as a neutrino source in a very long baseline oscillation experiment. In preparing for a talk in honor of Fred Reines a few months ago, I ran across a long forgotten 1972 letter from Bruno Pontecorvo, the originator of the hypothesis that oscillations may be observed in solar neutrino experiments. For your interest, I enclose a reproduction of this letter in Fig. 1.

For the benefit of neutrino pioneers of today, it is perhaps worth remarking that Ray Davis and I never considered the possibility that solar neutrinos could be used to learn more about neutrinos when, in the early 1960's, we were first analyzing the potentialities of a practical chlorine experiment. We sold the experiment as a fundamental test of the hypothesis that the sun shines by nuclear fusion reactions in its interior. Only after



ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ
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Prof. J.N.Bahcall
The Institute for Advanced Study
School of Natural Science
Princeton, New Jersey 08540, USA

Dear Prof. Bahcall,

Thank you very much for your letter and the abstract of the new Davis investigation the numerical results of which I did not know. It starts to be really interesting! It would be nice if all this will end with something unexpected from the point of view of particle physics. Unfortunately, it will not be easy to demonstrate this, even if nature works that way.

I will attend the Balaton meeting on neutrinos and looking forward to see you there.

Yours sincerely,

B.Pontecorvo

BMP/nn

Figure 1. Letter from Bruno Pontecorvo in 1972.

the first results of the chlorine experiment showed in 1968 a conflict with the solar model calculations and Gribov and Pontecorvo published their epochal 1969 paper on vacuum oscillations of solar neutrinos did we begin to consider the possibility that solar neutrinos might tell us something new about particle physics. Maybe there are previously unimagined physics treasures to be discovered in future neutrino experiments.

2. Standard Model Predictions

Table 1 gives the neutrino fluxes and their uncertainties for our best standard solar model, hereafter BP98. [10] Figure 2 shows the predicted neutrino fluxes from the dominant p-p fusion chain.

The BP98 solar model includes diffusion of heavy elements and helium, makes use of the nuclear reaction rates recommended by the expert workshop held at the Institute of Nuclear Theory, [12] recent (1996) Livermore OPAL radiative opacities, [13] the OPAL equation of state, [14]

Table 1
Standard Model Predictions (BP98): solar neutrino fluxes and neutrino capture rates, with 1σ uncertainties from all sources (combined quadratically).

Source	Flux $\left(10^{10}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}\right)$	Cl (SNU)	Ga (SNU)
pp	$5.94\left(1.00^{+0.01}_{-0.01}\right)$	0.0	69.6
pep	$1.39\times10^{-2}\left(1.00^{+0.01}_{-0.01}\right)$	0.2	2.8
hep	2.10×10^{-7}	0.0	0.0
^{7}Be	$4.80\times10^{-1}\left(1.00^{+0.09}_{-0.09}\right)$	1.15	34.4
^{8}B	$5.15\times10^{-4}\left(1.00^{+0.19}_{-0.14}\right)$	5.9	12.4
^{13}N	$6.05\times10^{-2}\left(1.00^{+0.19}_{-0.13}\right)$	0.1	3.7
^{15}O	$5.32\times10^{-2}\left(1.00^{+0.22}_{-0.15}\right)$	0.4	6.0
^{17}F	$6.33\times10^{-4}\left(1.00^{+0.12}_{-0.11}\right)$	0.0	0.1
Total		$7.7^{+1.2}_{-1.0}$	129^{+8}_{-6}

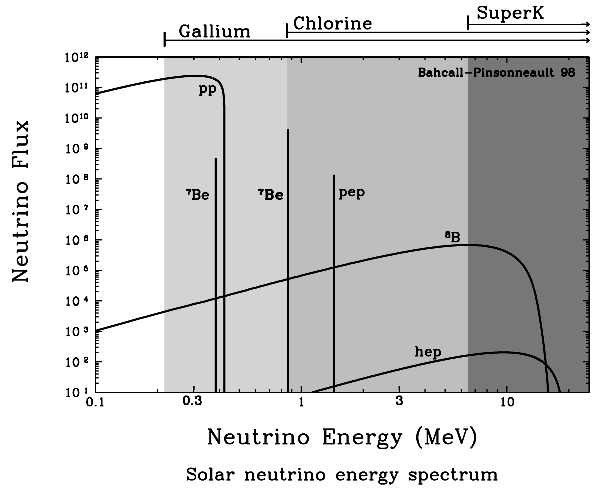

Figure 2. The energy Spectrum of neutrinos from the pp chain of interactions in the Sun, as predicted by the standard solar model. Neutrino fluxes from continuum sources (such as pp and 8B) are given in the units of counts per cm2 per second. The pp chain is responsible for more than 98% of the energy generation in the standard solar model. Neutrinos produced in the carbon-nitrogen-oxygen CNO chain are not important energetically and are difficult to detect experimentally. The arrows at the top of the figure indicate the energy thresholds for the ongoing neutrino experiments.

and electron and ion screening as determined by the recent density matrix calculation. [15,16] The neutrino absorption cross sections that are used in constructing Table 1 are the most accurate values available [17,18] and include, where appropriate, the thermal energy of fusing solar ions and improved nuclear and atomic data. The validity of the absorption cross sections has recently been confirmed experimentally using intense radioactive sources of ^{51}Cr. The ratio, R, of the capture rate measured (in GALLEX and SAGE) to the calculated ^{51}Cr capture rate is $R = 0.95 \pm 0.07$ (exp) + $^{+0.04}_{-0.03}$ (theory) and was discussed extensively at Neutrino 98 by Gavrin and by Kirsten. The neutrino-electron scattering cross sections, used in interpreting the Kamiokande and SuperKamiokande experiments, now include electroweak radiative corrections. [19]

Figure 3 shows for the chlorine experiment all the predicted rates and the estimated uncertainties (1σ) published by my colleagues and myself since the first measurement by Ray Davis and his colleagues in 1968. This figure should give you some feeling for the robustness of the solar model calculations. Many hundreds and probably thousands of researchers have, over three decades, made great improvements in the input data for the solar models, including nuclear cross sections, neutrino cross sections, measured element abundances on the surface of the sun, the solar luminosity, the stellar radiative opacity, and the stellar equation of state. Nevertheless, the most accurate predictions of today are essentially the same as they were in 1968 (although now they can be made with much greater confidence). For the gallium experiments, the neutrino fluxes predicted by standard solar models, corrected for diffusion, have been in the range 120 SNU to 141 SNU since 1968. [17] A SNU is a convenient unit with which to describe the measured rates of so-

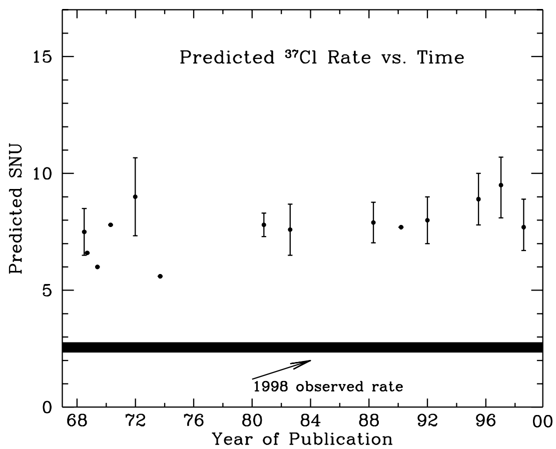

Figure 3. The predictions of John Bahcall and his collaborators of neutrino capture rates in the ^{37}Cl experiment are shown as a function of the date of publication (since the first experimental report in 1968. [1]) The event rate SNU is a convenient product of neutrino flux times interaction cross section, 10^{-36} interactions per target atom per sec. The format is from Figure 1.2 of the book Neutrino Astrophysics. [9] The predictions have been updated through 1998.

lar neutrino experiments: 10^{-36} interactions per target atom per second.

There are three reasons that the theoretical calculations of neutrino fluxes are robust: 1) the availability of precision measurements and precision calculations of input data; 2) the connection between neutrino fluxes and the measured solar luminosity; and 3) the measurement of the helioseismological frequencies of the solar pressure-mode (p-mode) eigenfrequencies. I have discussed these reasons in detail in another talk. [20]

Figure 4 displays the calculated ^{7}Be and ^{8}B neutrino fluxes for all 19 standard solar models which have been published in the last 10 years in refereed science journals. The fluxes are normalized by dividing each published value by the flux from the BP98 solar model; [10] the abscissa is the normalized ^{8}B flux and the ordinate is the normalized ^{7}Be neutrino flux. The rectangular box shows the estimated 3σ uncertainties in the predictions of the BP98 solar model.

All of the solar model results from different groups fall within the estimated 3σ uncertainties in the BP98 analysis (with the exception of the Dar-Shaviv model whose results have not been reproduced by other groups). This agreement demonstrates the robustness of the predictions since the calculations use different computer codes (which achieve varying degrees of precision) and involve a variety of choices for the nuclear

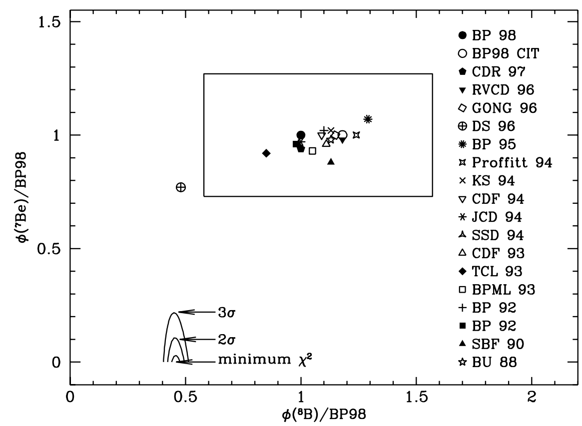

Figure 4. Predictions of standard solar models since 1988. This figure, which is Fig. 1 of Bahcall, Krastev and Smirnov (1998)[11], shows the predictions of 19 standard solar models in the plane defined by the ^{7}Be and ^{8}B neutrino fluxes. The abbreviations that are used in the figure to identify different solar models are defined in the bibliographical item, Ref. [21]. The figure includes all standard solar models with which I am familiar that were published in refereed journals in the decade 1988-1998. All of the fluxes are normalized to the predictions of the Bahcall-Pinsonneault 1998 solar model, BP98. [10] The rectangular error box defines the 3σ error range of the BP98 fluxes. The best-fit ^{7}Be neutrino flux is negative. At the 99% C.L., there is no solution [11] with all positive neutrino fluxes (see discussion in Section 6). All of the standard model solutions lie far from the best-fit solution, even far from the 3σ contour.

parameters, the equation of state, the stellar radiative opacity, the initial heavy element abundances, and the physical processes that are included.

The largest contributions to the dispersion in values in Figure 4 are due to the choice of the normalization for S_{17} (the production cross-section factor for ^{8}B neutrinos) and the inclusion, or non-inclusion, of element diffusion in the stellar evolution codes. The effect in the plane of Fig. 4 of the normalization of S_{17} is shown by the difference between the point for BP98 (1.0,1.0), which was computed using the most recent recommended normalization, [12] and the point at (1.18,1.0) which corresponds to the BP98 result with the earlier (CalTech) normalization. [22]

Helioseismological-observations
have shown [10,23] that element diffusion is occurring and must be included in solar models, so that the most recent models shown in Fig. 4 now all include helium and heavy element diffusion. By comparing a large number of earlier models, it was shown that all published standard solar models give the same results for solar neutrino fluxes to an accuracy of better than 10% if the same input parameters and physical processes are included. [24,25]

Bahcall, Krastev, and Smirnov [11] have compared the observed rates with the calculated, standard model values, combining quadratically the theoretical solar model and experimental uncertainties, as well as the uncertainties in the neutrino cross sections. Since the GALLEX and SAGE experiments measure the same quantity, we treat the weighted average rate in gallium as one experimental number. We adopt the Super-Kamiokande measurement as the most precise direct determination of the higher-energy ^{8}B neutrino flux.

Using the predicted fluxes from the BP98 model, the χ^2 for the fit to the three experimental rates (chlorine, gallium, and SuperKamiokande, see Fig. 5) is

$$\chi^2_{\rm SSM}(3 \text{ experimental rates}) = 61 \ . \qquad (1)$$

The result given in Eq. (1), which is approximately equivalent to a 20σ discrepancy, is a quantitative expression of the fact that the standard model predictions do not fit the observed solar neutrino measurements.

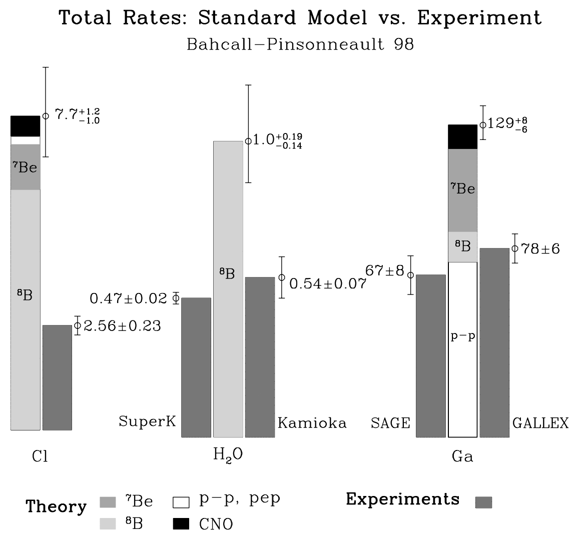

Figure 5. Comparison of measured rates and standard-model predictions for five solar neutrino experiments. [2–6] The unit for the radiochemical experiments (chlorine and gallium) is SNU (see Fig. 3 for a definition); the unit for the water-Cerenkov experiments (Kamiokande and Super-Kamiokande) is the rate predicted by the standard solar model plus standard electroweak theory. [10]

3. Three Solar Neutrino Problems

I will now compare the predictions of the combined standard model with the results of the operating solar neutrino experiments.

We will see that this comparison leads to three different discrepancies between the calculations and the observations, which I will refer to as the three solar neutrino problems.

Figure 5 shows the measured and the calculated event rates in the five ongoing solar neutrino experiments. This figure reveals three discrepancies between the experimental results and the expectations based upon the combined standard model. As we shall see, only the first of these discrepancies depends in an important way upon the pre-

dictions of the standard solar model.

3.1. Calculated versus Observed Absolute Rate

The first solar neutrino experiment to be performed was the chlorine radiochemical experiment, [2] which detects electron-type neutrinos that are more energetic than 0.81 MeV. After more than a quarter of a century of operation of this experiment, the measured event rate is 2.56 ± 0.23 SNU, which is a factor of three less than is predicted by the most detailed theoretical calculations, $7.7^{+1.2}_{-1.0}$ SNU. [10] Most of the predicted rate in the chlorine experiment is from the rare, high-energy ^{8}B neutrinos, although the ^{7}Be neutrinos are also expected to contribute significantly. According to standard model calculations, the *pep* neutrinos and the CNO neutrinos (for simplicity not discussed here) are expected to contribute less than 1 SNU to the total event rate.

This discrepancy between the calculations and the observations for the chlorine experiment was, for more than two decades, the only solar neutrino problem. I shall refer to the chlorine disagreement as the "first" solar neutrino problem.

3.2. Incompatibility of Chlorine and Water Experiments

The second solar neutrino problem results from a comparison of the measured event rates in the chlorine experiment and in the Japanese pure-water experiments, Kamiokande [3] and SuperKamiokande. [6] The water experiments detect higher-energy neutrinos, most easily above 7 MeV, by by observing the Cerenkov radiation from neutrino-electron scattering: $\nu + e \longrightarrow \nu' + e'$. According to the standard solar model, ^{8}B beta decay, and possibly the *hep* reaction, [26] are the only important source of these higher-energy neutrinos.

The Kamiokande and SuperKamiokande experiments show that the observed neutrinos come from the sun. The electrons that are scattered by the incoming neutrinos recoil predominantly in the direction of the sun-earth vector; the relativistic electrons are observed by the Cerenkov radiation they produce in the water detector. In addition, the water Cerenkov experiments measure the energies of individual scattered electrons and therefore provide information about the energy spectrum of the incident solar neutrinos.

The total event rate in the water experiments, about 0.5 the standard model value (see Fig. 5), is determined by the same high-energy ^{8}B neutrinos that are expected, on the basis of the combined standard model, to dominate the event rate in the chlorine experiment. I have shown elsewhere [27] that solar physics changes the shape of the ^{8}B neutrino spectrum by less than 1 part in 10^5 . Therefore, we can calculate the rate in the chlorine experiment (threshold 0.8 MeV) that is produced by the ^{8}B neutrinos observed in the Kamiokande and SuperKamiokande experiments at an order of magnitude higher energy threshold.

If no new physics changes the shape of the ^{8}B neutrino energy spectrum, the chlorine rate from ^{8}B alone is 2.8 ± 0.1 SNU for the SuperKamiokande normalization (3.2 ± 0.4 SNU for the Kamiokande normalization), which exceeds the total observed chlorine rate of 2.56 ± 0.23 SNU.

Comparing the rates of the SuperKamiokande and the chlorine experiments, one finds–assuming that the shape of the energy spectrum of ^{8}B ν_e's is not changed by new physics–that the net contribution to the chlorine experiment from the *pep*, ^{7}Be, and CNO neutrino sources is negative: -0.2 ± 0.3 SNU. The contributions from the *pep*, ^{7}Be, and CNO neutrinos would appear to be completely missing; the standard model prediction for the combined contribution of *pep*, ^{7}Be, and CNO neutrinos is a relatively large 1.8 SNU (see Table 1). On the other hand, we know that the ^{7}Be neutrinos must be created in the sun since they are produced by electron capture on the same isotope (^{7}Be) which gives rise to the ^{8}B neutrinos by proton capture.

Hans Bethe and I pointed out [28] that this apparent incompatibility of the chlorine and water-Cerenkov experiments constitutes a "second" solar neutrino problem that is almost independent of the absolute rates predicted by solar models. The inference that is usually made from this comparison is that the energy spectrum of ^{8}B neutrinos is changed from the standard shape by physics not included in the simplest version of

the standard electroweak model.

3.3. Gallium Experiments: No Room for ^{7}Be Neutrinos

The results of the gallium experiments, GALLEX and SAGE, constitute the third solar neutrino problem. The average observed rate in these two experiments is 73 ± 5 SNU, which is accounted for in the standard model by the theoretical rate of 72.4 SNU that is calculated to come from the basic *p-p* and *pep* neutrinos (with only a 1% uncertainty in the standard solar model *p-p* flux). The ^{8}B neutrinos, which are observed above 6.5 MeV in the Kamiokande experiment, must also contribute to the gallium event rate. Using the standard shape for the spectrum of ^{8}B neutrinos and normalizing to the rate observed in Kamiokande, ^{8}B contributes another 6 SNU. (The contribution predicted by the standard model is 12 SNU, see Table 1.) Given the measured rates in the gallium experiments, there is no room for the additional 34 ± 3 SNU that is expected from ^{7}Be neutrinos on the basis of standard solar models (see Table 1).

The seeming exclusion of everything but *p-p* neutrinos in the gallium experiments is the "third" solar neutrino problem. This problem is essentially independent of the previously-discussed solar neutrino problems, since it depends strongly upon the *p-p* neutrinos that are not observed in the other experiments and whose theoretical flux can be calculated accurately.

The missing ^{7}Be neutrinos cannot be explained away by a change in solar physics. The ^{8}B neutrinos that are observed in the Kamiokande experiment are produced in competition with the missing ^{7}Be neutrinos; the competition is between electron capture on ^{7}Be versus proton capture on ^{7}Be. Solar model explanations that reduce the predicted ^{7}Be flux generically reduce much more (too much) the predictions for the observed ^{8}B flux.

The flux of ^{7}Be neutrinos, $\phi(^7\mathrm{Be})$, is independent of measurement uncertainties in the cross section for the nuclear reaction $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$; the cross section for this proton-capture reaction is the most uncertain quantity that enters in an important way in the solar model calculations. The flux of ^{7}Be neutrinos depends upon the proton-capture reaction only through the ratio

$$\phi(^7\mathrm{Be}) \ \propto \ \frac{R(e)}{R(e)+R(p)}, \qquad (2)$$

where $R(e)$ is the rate of electron capture by ^{7}Be nuclei and $R(p)$ is the rate of proton capture by ^{7}Be. With standard parameters, solar models yield $R(p) \approx 10^{-3} R(e)$. Therefore, one would have to increase the value of the $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ cross section by more than two orders of magnitude over the current best-estimate (which has an estimated experimental uncertainty of $\sim 10\%$) in order to affect significantly the calculated ^{7}Be solar neutrino flux. The required change in the nuclear physics cross section would also increase the predicted neutrino event rate by more than 100 in the Kamiokande experiment, making that prediction completely inconsistent with what is observed.

I conclude that either: 1) at least three of the five operating solar neutrino experiments (the two gallium experiments plus either chlorine or the two water Cerenkov experiments, Kamiokande and SuperKamiokande) have yielded misleading results, or 2) physics beyond the standard electroweak model is required to change the energy spectrum of ν_e after the neutrinos are produced in the center of the sun.

4. Uncertainties in the Flux Calculations

I will now discuss uncertainties in the solar model flux calculations.

Table 2 summarizes the uncertainties in the most important solar neutrino fluxes and in the Cl and Ga event rates due to different nuclear fusion reactions (the first four entries), the heavy element to hydrogen mass ratio (Z/X), the radiative opacity, the solar luminosity, the assumed solar age, and the helium and heavy element diffusion coefficients. The $^{14}\mathrm{N}+p$ reaction causes a 0.2% uncertainty in the predicted pp flux and a 0.1 SNU uncertainty in the Cl (Ga) event rates.

The predicted event rates for the chlorine and gallium experiments use recent improved calculations of neutrino absorption cross sections. [17,18] The uncertainty in the prediction for the gallium rate is dominated by uncertainties in the neutrino

Table 2
Average uncertainties in neutrino fluxes and event rates due to different input data. The flux uncertainties are expressed in fractions of the total flux and the event rate uncertainties are expressed in SNU. The ^{7}Be electron capture rate causes an uncertainty of $\pm 2\%$ [29] that affects only the ^{7}Be neutrino flux. The average fractional uncertainties for individual parameters are shown.

<Fractional uncertainty>	pp 0.017	^{3}He^{3}He 0.060	^{3}He^{4}He 0.094	$^7\mathrm{Be}+p$ 0.106	Z/X 0.033	opac see text	lum 0.004	age 0.004	diffuse 0.15
Flux									
pp	0.002	0.002	0.005	0.000	0.002	0.003	0.003	0.0	0.003
^{7}Be	0.0155	0.023	0.080	0.000	0.019	0.028	0.014	0.003	0.018
^{8}B	0.040	0.021	0.075	0.105	0.042	0.052	0.028	0.006	0.040
SNUs									
Cl	0.3	0.2	0.5	0.6	0.3	0.4	0.2	0.04	0.3
Ga	1.3	0.9	3.3	1.3	1.6	1.8	1.3	0.20	1.5

absorption cross sections, +6.7 SNU (7% of the predicted rate) and −3.8 SNU (3% of the predicted rate). The uncertainties in the chlorine absorption cross sections cause an error, ± 0.2 SNU (3% of the predicted rate), that is relatively small compared to other uncertainties in predicting the rate for this experiment. For non-standard neutrino energy spectra that result from new neutrino physics, the uncertainties in the predictions for currently favored solutions (which reduce the contributions from the least well-determined ^{8}B neutrinos) will in general be less than the values quoted here for standard spectra and must be calculated using the appropriate cross section uncertainty for each neutrino energy. [17,18]

The nuclear fusion uncertainties in Table 2 were taken from Adelberger et al., [12] the neutrino cross section uncertainties from Bahcall (1997)[17] and Bahcall et al. (1996),[18] the heavy element uncertainty was taken from helioseismological measurements, [30] the luminosity and age uncertainties were adopted from BP95, [25] the 1σ fractional uncertainty in the diffusion rate was taken to be 15%, [31] which is supported by helioseismological evidence, [23] and the opacity uncertainty was determined by comparing the results of fluxes computed using the older Los Alamos opacities with fluxes computed using the modern Livermore opacities. [24]

To include the effects of asymmetric errors, the now publicly-available code for calculating rates and uncertainties (see discussion in previous section) was run with different input uncertainties and the results averaged. The software contains a description of how each of the uncertainties listed in Table 2 were determined and used.

The low energy cross section of the $^7\mathrm{Be}+p$ reaction is the most important quantity that must be determined more accurately in order to decrease the error in the predicted event rates in solar neutrino experiments. The ^{8}B neutrino flux that is measured by the Kamiokande, [3] Super-Kamiokande, [6] and SNO [32] experiments is, in all standard solar model calculations, directly proportional to the $^7\mathrm{Be}+p$ cross section. If the 1σ uncertainty in this cross section can be reduced by a factor of two to 5%, then it will no longer be the limiting uncertainty in predicting the crucial ^{8}B neutrino flux (cf. Table 2).

5. How Large an Uncertainty Does Helioseismology Suggest?

Could the solar model calculations be wrong by enough to explain the discrepancies between predictions and measurements for solar neutrino experiments? Helioseismology, which confirms predictions of the standard solar model to high precision, suggests that the answer is probably "No."

Figure 6 shows the fractional differences between the most accurate available sound speeds measured by helioseismology [33] and sound speeds calculated with our best solar model (with no free parameters). The horizontal line corresponds to the hypothetical case in which the model predictions exactly match the observed values. The rms fractional difference between the calculated and the measured sound speeds is 1.1×10^{-3} for the entire region over which the sound speeds are measured, $0.05R_\odot < R < 0.95R_\odot$. In the solar core, $0.05R_\odot < R < 0.25R_\odot$ (in which about 95% of the solar energy and neutrino flux is produced in a standard model), the rms fractional difference between measured and calculated sound speeds is 0.7×10^{-3}.

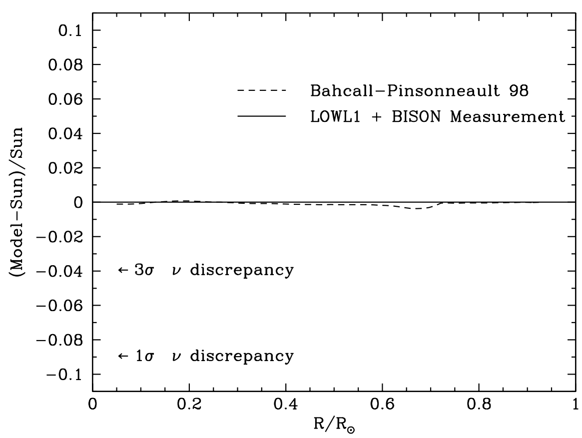

Figure 6. Predicted versus Measured Sound Speeds. This figure shows the excellent agreement between the calculated (solar model BP98, Model) and the measured (Sun) sound speeds, a fractional difference of 0.001 rms for all speeds measured between $0.05R_\odot$ and $0.95R_\odot$. The vertical scale is chosen so as to emphasize that the fractional error is much smaller than generic changes in the model, 0.04 to 0.09, that might significantly affect the solar neutrino predictions.

Helioseismological measurements also determine two other parameters that help characterize the outer part of the sun (far from the inner region in which neutrinos are produced): the depth of the solar convective zone (CZ), the region in the outer part of the sun that is fully convective, and the present-day surface abundance by mass of helium ($Y_{\rm surf}$). The measured values, $R_{\rm CZ} = (0.713 \pm 0.001)R_\odot$, [34] and $Y_{\rm surf} = 0.249 \pm 0.003$, [30] are in satisfactory agreement with the values predicted by the solar model BP98, namely, $R_{\rm CZ} = 0.714R_\odot$, and $Y_{\rm surf} = 0.243$. However, we shall see below that precision measurements of the sound speed near the transition between the radiative interior (in which energy is transported by radiation) and the outer convective zone (in which energy is transported by convection) reveal small discrepancies between the model predictions and the observations in this region.

If solar physics were responsible for the solar neutrino problems, how large would one expect the discrepancies to be between solar model predictions and helioseismological observations? The characteristic size of the discrepancies can be estimated using the results of the neutrino experiments and scaling laws for neutrino fluxes and sound speeds.

All recently published solar models predict essentially the same fluxes from the fundamental pp and pep reactions (amounting to 72.4 SNU in gallium experiments, cf. Table 1), which are closely related to the solar luminosity. Comparing the measured gallium rates and the standard predicted rate for the gallium experiments, the ^{7}Be flux must be reduced by a factor N if the disagreement is not to exceed n standard deviations, where N and n satisfy $72.4 + (34.4)/N = 72.2 + n\sigma$. For a 1σ (3σ) disagreement, $N = 6.1(2.05)$. Sound speeds scale like the square root of the local temperature divided by the mean molecular weight and the ^{7}Be neutrino flux scales approximately as the 10th power of the temperature. [35] Assuming that the temperature changes are dominant, agreement to within 1σ would require fractional changes of order 0.09 in sound speeds (3σ could be reached with 0.04 changes), if all model changes were in the temperature[1]. This argu-

[1] I have used in this calculation the GALLEX and SAGE

ment is conservative because it ignores the ^{8}B and CNO neutrinos which contribute to the observed counting rate (cf. Table 1) and which, if included, would require an even larger reduction of the ^{7}Be flux.

I have chosen the vertical scale in Fig. 6 to be appropriate for fractional differences between measured and predicted sound speeds that are of order 0.04 to 0.09 and that might therefore affect solar neutrino calculations. Fig. 6 shows that the characteristic agreement between solar model predictions and helioseismological measurements is more than a factor of 40 better than would be expected if there were a solar model explanation of the solar neutrino problems.

6. Fits Without Solar Models

Suppose (following the precepts of Hata et al., [36] Parke, [37] and Heeger and Robertson [38]) we now ignore everything we have learned about solar models over the last 35 years and allow the important *pp*, ^{7}Be, and ^{8}B fluxes to take on any non-negative values. What is the best fit that one can obtain to the solar neutrino measurements assuming only that the luminosity of the sun is supplied by nuclear fusion reactions among light elements (the so-called 'luminosity constraint')? [39]

The answer is that the fits are bad, even if we completely ignore what we know about the sun. I quote the results from Bahcall, Krastev and Smirnov (1998).

If the CNO neutrino fluxes are set equal to zero, there are no acceptable solutions at the 99% C. L. ($\sim 3\sigma$ result). The best-fit is worse if the CNO fluxes are not set equal to zero. All so-called 'solutions' of the solar neutrino problems in which the astrophysical model is changed arbitrarily (ignoring helioseismology and other constraints) are inconsistent with the observations at much more than a 3σ level of significance. No fiddling of the physical conditions in the model can yield the minimum value, quoted above, that was found by varying the fluxes independently and arbitrarily.

Figure 4 shows, in the lower left-hand corner, the best-fit solution and the 1σ –3σ contours. The 1σ and 3σ limits were obtained by requiring that $\chi^2 = \chi^2_{\rm min} + \delta\chi^2$, where for 1σ $\delta\chi^2 = 1$ and for 3σ $\delta\chi^2 = 9$. All of the standard model solutions lie far from the best-fit solution and even lie far from the 3σ contour.

Since standard model descriptions do not fit the solar neutrino data, we will now consider models in which neutrino oscillations change the shape of the neutrino energy spectra.

Table 3
Neutrino Oscillation Solutions.

Solution	Δm^2	$\sin^2 2\theta$
SMA	5×10^{-6} eV^2	5×10^{-3}
LMA	2×10^{-5} eV^2	0.8
LOW	8×10^{-8} eV^2	0.96
VAC	8×10^{-11} eV^2	0.7

7. Neutrino Oscillations

The experimental results from all five of the operating solar neutrino experiments (chlorine, Kamiokande, SAGE, GALLEX, and Super-Kamiokande) can be fit well by descriptions involving neutrino oscillations, either vacuum oscillations (as originally suggested by Gribov and Pontecorvo [40])or resonant matter oscillations (as originally discussed by Mikheyev, Smirnov, and Wolfenstein (MSW) [41]).

Table 3 summarizes the four best-fit solutions that are found in the two-neutrino approximation. [11,26] Only the SMA and vacuum oscillation solutions fit well the recoil electron energy spectrum measured in the Super-Kamiokande experiment–if the standard value for the *hep* production reaction cross section (^{3}He + p $\rightarrow$ ^{4}He + e^+ + ν_e) is used. [11] However, for over a decade I have not given an estimated uncertainty for this cross section. [9] The

measured rates reported by Kirsten and Gavrin at Neutrino 98. The experimental rates used in BP98 were not as precise and therefore resulted in slightly less stringent constraints than those imposed here. In BP98, we found that agreement to within 1σ with the then available experimental numbers would require fractional changes of order 0.08 in sound speeds (3σ could be reached with 0.03 changes.)

transition matrix element is essentially forbidden and the actual quoted value for the production cross section depends upon a delicate cancellation between two comparably sized terms that arise from very different and hard to evaluate nuclear physics. I do not see anyway at present to determine from experiment or from first principles theoretical calculations a relevant, robust upper limit to the *hep* production cross section (and therefore the *hep* solar neutrino flux).

The possible role of *hep* neutrinos in solar neutrino experiments is discussed extensively in Bahcall and Krastev (1998) [26] The most important unsolved problem in theoretical nuclear physics related to solar neutrinos is the range of values allowed by fundamental physics for the *hep* production cross section.

8. Discussion

When the chlorine solar neutrino experiment was first proposed, [42] the only stated motivation was "...to see into the interior of a star and thus verify directly the hypothesis of nuclear energy generation in stars." This goal has now been achieved,

The focus has shifted to using solar neutrino experiments as a tool for learning more about the fundamental characteristics of neutrinos as particles. Experimental effort is now concentrated on answering the question: What are the probabilities for transforming a solar ν_e of a definite energy into the other possible neutrino states? Once this question is answered, we can calculate what happens to ν_e's that are created in the interior of the sun. Armed with this information from weak interaction physics, we can return again to the original motivation of using neutrinos to make detailed, quantitative tests of nuclear fusion rates in the solar interior. Measurements of the flavor content of the dominant low energy neutrino sources, *p*-*p* and ^{7}Be neutrinos, will be crucial in this endeavor and will require another generation of superb solar neutrino experiments (see the comments in Section 9).

Three decades of refining the input data and the solar model calculations has led to a predicted standard model event rate for the chlorine experiment, 7.7 SNU, which is very close to 7.5 SNU, the best-estimate value obtained in 1968. [8] The situation regarding solar neutrinos is, however, completely different now, thirty years later. Four experiments have confirmed the original chlorine detection of solar neutrinos. Helioseismological measurements are in excellent agreement with the standard solar model predictions and very strongly disfavor (by a factor of 40 or more) hypothetical deviations from the standard model that are require to fits the neutrino data (cf. Fig. 6). Just in the last two years, improvements in the helioseismological measurements have resulted in a five-fold improvement in the agreement between the calculated standard solar model sound speeds and the measured solar velocities (cf. Figure 2 of the Neutrino 96 talk [43] with Figure 6 of this talk).

9. What next?

More than 98% of the calculated standard model solar neutrino flux lies below 1 MeV. The rare ^{8}B neutrino flux is the only solar neutrino source for which measurements of the energy have been made, but ^{8}B neutrinos constitute a fraction of less than 10^{-4} of the total solar neutrino flux.

The next goal of solar neutrino astronomy is to measure neutrino fluxes below 1 MeV. We should begin today preparing for experiments that will measure the ^{7}Be neutrinos (energy of 0.86 MeV) and the fundamental *p*-*p* neutrinos (< 0.43 MeV). Indeed, we have heard at this workshop some marvelously exciting descriptions of how such low energy experiments could be carried out. The BOREXINO observatory, which can detect $\nu - e$ scattering, is the only approved solar neutrino experiment which can measure energies less than 1 MeV.

The *p*-*p* neutrinos are overwhelmingly the most abundant source of solar neutrinos, carrying about 91% of the total flux according to the standard solar model. The ^{7}Be neutrinos constitute about 7% of the total standard model flux.

If we want to test and to understand neutrino oscillations with high precision using solar neutrino sources, then we have to measure the neutrino-electron scattering rate with ^{7}Be neutri-

nos, as will be done with the BOREXINO experiment, and also the CC (neutrino-absorption) rate with ^{7}Be neutrinos (no approved experiment). With a neutrino line as provided by ^{7}Be electron-capture in the sun, unique and unambiguous tests of neutrino oscillation models can be carried out if one knows both the charged-current and the neutral current reaction rates [44].

I believe we have calculated the flux of *p*-*p* neutrinos produced in the sun to an accuracy of $\pm 1\%$. Unfortunately, we do not yet have a direct measurement of this flux. The gallium experiments only tell us the rate of capture of all neutrinos with energies above 0.23 MeV.

The most urgent need for solar neutrino research is to develop a practical experiment to measure directly the *p*-*p* neutrino flux and the energy spectrum of electrons produced by target interactions with *p*-*p* neutrinos. Such an experiment can be used to test the precise and fundamental standard solar model prediction of the *p*-*p* neutrino flux. Moreover, the currently favored neutrino oscillation solutions all predict a strong influence of oscillations on the low-energy flux of ν_e.

Figure 7 shows the calculated neutrino survival probability as a function of energy for three global best-fit MSW oscillation solutions. You can see directly from this figure why we have to have accurate measurements for the *p*-*p* and ^{7}Be neutrinos: the currently favored solutions exhibit their most characteristic and strongly energy dependent features below 1 MeV. In all of these solutions, the survival probability shows a dramatic increase with energy below 1 MeV, whereas in the region above 5 MeV (accessible to SuperKamiokande and to SNO) the energy dependence of the survival probability is at best modest.

The *p*-*p* neutrinos are the gold ring of solar neutrino astronomy. Their measurement will constitute a simultaneous and critical test of stellar evolution theory and of neutrino oscillation solutions.

The most exciting result of this workshop for me has been the possibility discussed here of a synergistic experiment involving a huge (megaton?) nucleon decay detector in which an inner region is reserved for solar neutrino experiments(see, for example, the talks by Jung, Nakahata, and Ypsilantis at this workshop). The most straightforward solar neutrino experiments that could be carried out with this detector would be precision measurements of the temporal dependences of the relatively high-energy ^{8}B neutri-

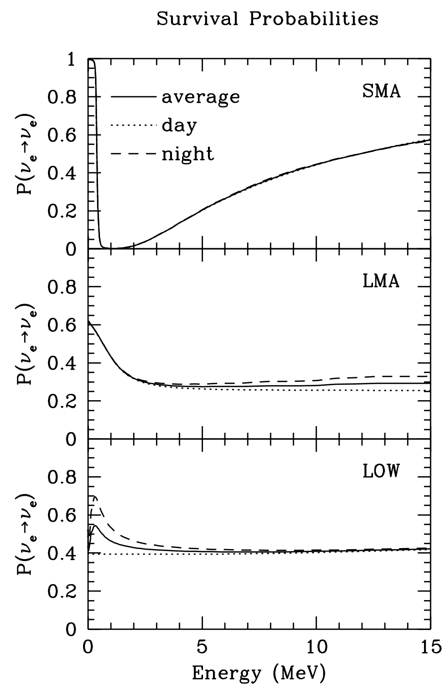

Figure 7. Survival probabilities for MSW solutions. The figure presents the yearly-averaged survival probabilities for an electron neutrino that is created in the sun to remain an electron neutrino upon arrival at the SuperKamiokande detector. There are only slight differences between the computed regeneration probabilities for the detectors located at the positions of Super-Kamiokande, SNO and the Gran Sasso Underground Laboratory. The full line refers to the average survival probabilities computed taking into account regeneration in the earth and the dotted line refers to calculations for the daytime that do not include regeneration. The dashed line includes regeneration at night. This is Fig. 9 of the 1998 analysis by Bahcall, Krastev, and Smirnov [11].

nos. One could measure with such a detector the zenith-angle dependence of the solar neutrino-event rate (the generalization of the day-night difference) and the seasonal dependence (generalization of the winter-summer difference). The design could relax somewhat the precise requirements for energy calibration and for energy resolution used for the SuperKamiokande and SNO experiments and concentrate instead on limiting the systematic uncertainties in the detector that could contribute to the error budget in the day-night or seasonal dependences. After three years of very careful measurements, the SuperKamiokande experiment has, as we have heard at this conference, about a 2σ result for the day-night difference. They do not yet have the statistics to report a meaningful measurement of the full zenith-angle dependence or the seasonal dependence. The predicted temporal effects are small, generally of order a percent, with the currently favored neutrino oscillation solutions.

Nature has provided us with many different baselines and with many different matter column densities with which to do Very Long Baseline (VLB) studies of neutrino oscillations. The earth-sun distance varies continuously during the year between $1.496(1.0 \pm 0.017)10^{13}$ cm and the column density through the earth to a terrestrial detector varies from 0 gm cm^{-2} during the day to more than 10^9 gm cm^{-2} at night.

A solar neutrino detector ten or more times the volume of the current SuperKamiokande experiment, as discussed in concept at this workshop, could measure precisely the results of many different VLB neutrino oscillation experiments. This would be a fantastic 'Smoking Gun' detector. I have a hard time sitting down when imagining such an exciting possibility.

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